Investing Choices and Risk Measures

In this part (starting from the previous chapter), we maintain the same assumptions:

- We assume perfect markets, so we assume four market features:
  1. No differences in opinion.
  2. No taxes.
  3. No transaction costs.
  4. No big sellers/buyers—we have infinitely many clones that can buy or sell.
- We already allow for unequal rates of returns in each period.
- We already allow for uncertainty. So, we do *not* know in advance what the rates of return on every project are.
- **But in contrast to Chapters 6, we no longer assume risk-neutrality. We are allowing for risk aversion now.**

  Recall Chapter 4, in which we found out that you are risk-averse.

- In this chapter, we lay the groundwork for understanding how investors choose among many different projects.
- You need this [a] to think as a manager about your company's investment risk; [b] more importantly to think about your "opportunity cost of capital," $\mathcal{E}(\tilde{r})$.

Some Particular Investments

Scenarios		Assets A	B	C	D
1/4	Yellow	-4	-1	-1.25	$+3$
1/4	Red	-4	$+9$	$+1.25$	$+13$
1/4	Green	$+6$	$+9$	$+3.75$	$+3$
1/4	Blue	$+6$	-1	$+1.25$	-7

We will be using these four assets (A to D) in these slides.

You can think of these as rates of return (or, if need be, as dollar returns).

(These numbers are intentionally different from those in the book.)

Preferences

Presume you can only hold one asset. Lets compare A to each B:

	State 1	State 2	State 3
Investment A	–5%	5%	15%
Investment B1	–6%	4%	10%
Investment B2	–6%	6%	5%
Investment B3	–10%	5%	20%
Investment B4	–10%	6%	20%

Q1: Would every investor prefer A to each B? Would you?

Measuring Risk and Reward

Use the next page for calculations.

Portfolio Reward:

Q2: What is the reward of your above investment opportunities?

Portfolio Risk:

Q3: What is the risk of your above investment opportunities?

Reminder: If these returns are just representative historical realizations from a population, you would divide by 3, not 4 in your computation of the variance.

Important: The standard deviation as a measure of risk applies only to your overall portfolio, not to individual securities.

	A	B	C	D	$A - \overline{A}$	$B - \overline{B}$	$C - \overline{C}$	$D - \overline{D}$
Yellow	-4	-1	-1.25	$+3$				
Red	-4	$+9$	$+1.25$	$+13$				
Green	$+6$	$+9$	$+3.75$	$+3$				
Blue	$+6$	-1	$+1.25$	-7				
Mean ($\mathcal{E}$)								
$\mathcal{V}ar$								
$\mathcal{S}dv$								

Diversification

Q4: What's the risk of an (equal-weighted) pfio of A and B?
(HINT: First compute the rates of returns of the combination portfolio in each state.)

Q5: Is the average portfolio or are the individual components riskier? Why?

Q6: What kind of portfolio would you—a smart investor—hold?

Q7: In real life, what sort of portfolios should smart investors hold?

Effects of Comovement

	A	B	C	D	Half A, half **C**	Half A, half **D**
Yellow	-4	-1	-1.25	$+3$		
Red	-4	$+9$	$+1.25$	$+13$		
Green	$+6$	$+9$	$+3.75$	$+3$		
Blue	$+6$	-1	$+1.25$	-7		
Mean ($\mathcal{E}$)						
$\mathcal{V}ar$						
$\mathcal{S}dv$						

Own Risk

Q8: Is C or D the riskier investment?

Q9: If you own A, is C or D the riskier addition?

Q10: Why?

Q11: If investors are smart, what is their A?

Q12: If you are selling to smart investors either C or D, for which of these two projects do you think will investors clamor to invest in your project (i.e., accept a lower expected rate of return)?

Important:

- The fundamental insight of investments: Investors care about overall portfolio risk, not about the constituent component risk.
- From a corporate managerial perspective, it is not your projects that are low risk in themselves that are highly desirable for your investors, but projects which wiggle opposite to the rest of their portfolios

Covariance, Beta, Correlation

Use the next page for calculations.

We need a measure of synchronicity of securities with our pfio.

Consider A as being M, which is our base (market) portfolio.

Q13: What kind of synchronicity (correlation or beta) of our project are we interested in?

Q14: How do you compute the three candidates: covariance, correlation, and beta?

Q15: What is the market-beta of the market (the S&P500)?

	M	B	C	D	$M - \overline{M}$	$B - \overline{B}$	$C - \overline{C}$	$D - \overline{D}$
Yellow	-4	-1	-1.25	$+3$				
Red	-4	$+9$	$+1.25$	$+13$				
Green	$+6$	$+9$	$+3.75$	$+3$				
Blue	$+6$	-1	$+1.25$	-7				
Mean ($\mathcal{E}$)								
$\mathcal{V}ar$								
$\mathcal{S}dv$								

Which One?

Consider A as being M, which is our base (market) portfolio.

- Covariance has uninterpretable units. Yuck.
- Correlation has a scale problem.

  The correlation would tell us that a security with rates of return $R = (0.9, 1.0, 1.1, 1.0)$ has the same 70.7% correlation with A as $S = 1000 \cdot R - 1000 = (-100, 0, +100, 0)$ does—but \$100 of R will clearly contribute less risk to our portfolio $\mathcal{M}$ than \$100 of S.

Therefore, we prefer measuring risk contribution of B or C by its market-beta with respect to A.

- Beta is similar to correlation. It always has the same sign.
- Beta can be interpreted as a slope. Put A (M) on the X axis, and your project B (or C) on the Y axis. A slope of 1 is a diagonal line. A slope of 0 is a horizontal line. A slope of ∞ is a vertical line.
- Without alpha, beta tells you how an x% higher rate of return (than normal) in the market will likely reflect itself simultaneously in a $\beta_i \cdot x\%$ higher rate of return in your stock.
- Together with alpha, beta can be interpreted as giving you the best conditional forecast of your project's rate of return, given a market outcome scenario's rate of return.

Estimating market-beta from historical data is discussed in 8-3D.

Preview of Equilibrium

Q16: Ceteris paribus, should/do investors prefer securities with a higher beta or a lower beta with respect to their (market) portfolio?

Q17: Should/do high beta or low beta projects have to offer higher average rates of return?

Q18: Should/do high variance or low variance projects have to offer higher average rates of return?

Also look at Yahoo!*Finance* for some market-betas of some common stocks.

Market-Beta Weighted Averaging

Q19: If you own a firm consisting of $4 million invested in Division C, and $6 million in Division D, what is the market beta of your firm? [$\beta_C = 0.25; \beta_D = -1$]

	M	M−$\overline{\text{M}}$	C	D	Firm	Fm − $\overline{\text{Fm}}$
Yellow	-4	-5	-1.25	$+3$		
Red	-4	-5	$+1.25$	$+13$		
Green	$+6$	$+5$	$+3.75$	$+3$		
Blue	$+6$	$+5$	$+1.25$	-7		
Mean ($\mathcal{E}$)	1	0	1.25	3		
$\mathcal{V}ar$	25	25	3.125	50		
$\beta_{i,M}$	1		0.25	–1		

Q20: How do you compute the overall market-beta of your firm, based on its projects?

Q21: What statistics can you “value-average”? What statistics can you not “value-average.”

Mostly omitted, but in the Appendix

The mean-variance efficient frontier (almost equivalently, the mean-standard deviation efficient frontier).

- Covering it would require a full lecture. Though omitted, the mean-variance efficient frontier is extremely important. It is the basis for modern finance and for the CAPM.
- The frontier gives you the optimal set of assets that you should hold if you want to tolerate a risk of x%. It also tells you what expected rate of return this portfolio (for your specific risk-tolerance) should give you.

- There is a medium painful (but common and important) formula for computing the overall variance of a portfolio, based on the variance-covariance between all assets, and your investment in each asset. This formula makes it easy to recompute the portfolio risk when you change portfolio holdings. Its simplest form is for a portfolio P with two assets, A and B:

$$\mathcal{V}ar(\tilde{R_P}) = \mathcal{V}ar(w_A \cdot \tilde{R_A} + w_B \cdot \tilde{R_B}) =$$

$$w_A^2 \cdot \mathcal{V}ar(\tilde{R}_A) + w_B^2 \cdot \mathcal{V}ar(\tilde{R}_B) + 2 \cdot w_A \cdot w_B \cdot \mathcal{C}ov(\tilde{R}_A, \tilde{R}_B)$$

  This formula also shows that value weighting variances does not work, because this expression is *not* $w_A \cdot \mathcal{V}ar(\tilde{R}_A) + w_B \cdot \mathcal{V}ar(\tilde{R}_B)$.)

- There is an extremely important application of this formula (in q&a), which you must know:
  - Rates of return over time are usually uncorrelated (or you could use past stock returns to outpredict future stock returns). Algebraically,

    $$\mathcal{C}ov(\tilde{R}_t, \tilde{R}_{t+i}) \approx 0$$

    where the subscripts t and $t+i$ refer to time periods, not to stocks (as our subscripts usually do).
  - Let's presume that you know that the per-unit-of-time standard deviation is constant. Let's just call this number σ.
  - In this case, the following approximation is not bad:

    $$\mathcal{S}dv(\tilde{R}_{0,T}) \approx \sqrt{T} \cdot \sigma$$

    For example, if your portfolio risk is 10% per month, then it is about $\sqrt{12} \cdot 10\% \approx 35\%$ per year.
  - This is also used in the Sharpe-ratio, a (badly flawed but common) measure of investment performance that divides the historical average rate of return (net of the risk-free rate) by its standard deviation. (The SR of a portfolio grows with the square-root of time.)

Homework Assignment

1. Reread Chapter 8.
2. Read Chapter 9.
3. Hand in all Chapter 8 end-of-chapter problems, due in 7 days.