Uncertainty, Default, and Risk

In this part (consisting of three chapters), we maintain the assumptions of the previous chapter:

- We assume perfect markets, so we assume four market features:
  1. No differences in opinion.
  2. No taxes.
  3. No transaction costs.
  4. No big sellers/buyers—we have infinitely many clones that can buy or sell.
- We already allow for unequal rates of returns in each period.
- **But we now allow uncertainty. So, we do *not* know in advance what the rates of return on every project are.**

We now need to predict (describe) the future. For this, we need statistics. Attend the rest of this course at your own risk:

> Persons pretending to forecast the future shall be considered disorderly under subdivision 3, section 901 of the criminal code and liable to a fine of $ 250 and/or six months in prison.
> * (Section 889, New York State Code of Criminal Procedure.)

Statistics and Random Variables

- Covered fully in your sadistics [sic] course.
- A **random variable** (denoted by a tilde over the variable) is not an ordinary variable.

  It can take on a whole range of possibilities, often drawn in a histogram. In a sense, it is more like a function of a randomizing device (e.g., a coin): it needs to obtain a value from this randomizing device. For example, you may get $10 if heads, $50 if tails.

  It is often useful to think of a random variable as the histogram itself. This is my intuition.

- In statistical applications, we usually *assume* that we know the histogram, but not the outcome.

Expected Value

- Think of the expected value (mean) of a random variable as the average if you repeat the experiment infinitely many times.
- From a RV's histogram, to obtain the expected value, multiply each outcome by its probability, and add 'em up. Example:

$$\text{A Die : ``1''} = -\$6; \text{``2''} = \$36; \text{``3''} = -\$12; \text{``4 to 6''} = \$150.$$

Q1: What is the expected payoff?

- The expectation is not a realization!!

- You can also compute the expectation of a function of random variables. For example,

Q2: What is the expected value of the die-squared?

Big Leap of Faith

- Assuming we know the histogram is good for a throw of a die, where we know the physics.
- We do not really know the histogram for the rate of return on the stock market.
- Therefore, we often use many historical outcome draws to proxy directly (or occasionally with some manipulation) for the true unknown histogram of future probabilities.
  - This translation of the historical outcome histogram (distribution) into the future outcome histogram (distribution) is a heroic assumption—but it is often the only reasonable information that we have. You have no better alternatives.
  - If we use the historical realized distribution, some statistics require a small correction. (This is the least of our problems.) Will tell you soon.
  - We know where this works well, and where this does not work well. You must be mindful of where it works and where it fails.
  - For example, what is the expected rate of return on the stock market? Is it the same that it was historically?

- If we assume the histogram is known, the expectations are known!
- We almost inevitably pretend we know the histogram—because we have no better alternatives. We will usually assume this heroically.

Fair Bets

Q3: What is a fair bet? What is it in the above die-bet example?

Variance

6-1B

- The variance is, roughly speaking, the **expected squared deviation from the mean**. REMEMBER THIS INTUITIVE NAME!
- Think of the true unknown variance of a random variable as the computed squared deviation from the mean if you draw infinitely many realizations.

  (In some other classes, you may also encounter the variance of a mean estimate. This is a related concept, but it is different. We won't use it.)

- From a RV's histogram, to obtain the variance, multiply each squared deviation from the mean by its probability, and add 'em up.

  Example:

State	Prob	Outcome	Outcome –Mean	Squared
"1"	1/6	–$6	–$84	$$7,056
"2"	1/6	$36	–$42	$$1,764
"3"	1/6	–$12	–$90	$$8,100
"4"-"6"	3/6	$150	+$72	$$5,184
Weighted Mean:		$78	$0	$$5,412

- The units on a variance are incomprehensible.

Standard Deviation

- The standard deviation is the square-root of the variance:

$$\text{SD} = \sqrt{\$\$5,412} \approx \$73.57$$

- Intuitively, think of the standard-deviation is the "typical deviation from the mean" of the next draw.

  (This is not entirely correct, but often helps a lot.)
- If a variance is higher, then the standard deviation is higher.
- When people talk about risk, they often use phrasing such as "if the variance is higher." They could equally well say "if the standard deviation is higher."
- This is economics in action:

  Variance has 2 syllables, standard deviation has 5 syllables.

Check your Knowledge:

Compute Statistics For Historical Returns

6-1

Q4: What is the expected mean and standard deviation if stock returns followed this historical distribution:

+10% –5% +20% +15%

Means and Variance Estimates For Stock Returns

- We use historical rate of return realizations to estimate the distribution of future rates of return.
- We assume each historical realization was an equally likely draw.
- When we compute the variance from a histogram that relies on a sample (not the known histogram of the population), we adjust by dividing not by N, but $N - 1$.

  Why Excel and Stats packages sometimes give different answers for Var and Sdv:
  We divide by N, because we presume you know the population. Excel and stats packages without the 'p' at the tail end of the function name divide by $N - 1$, because they presume you know only a sample drawn from the population. Usually, this makes little difference if you have reasonably large datasets—but it makes a difference in our small $N = 4$ datasets. So, you must use stdevp and varp, and not stdev and var to get the same results as those we compute in these slides in class.

- Let me warn you again: Do not trust the historical means blindly especially when it comes to predicting future expected rates of returns. For individual stocks (rather than big diversified portfolios), this would be incredibly bad. Even for big diversified portfolios, this is a big leap of faith.
- *In contrast, historical variances (and covariances and standard deviations) are usually reasonably good predictors of future variances (and covariances and standard deviations).*

  Advice: use a couple of years, say 3–5, of historical data to estimate them.

Risk Neutrality

- .

Q5: If I offer you a bet of +\$1 if heads and $-\$1$ if tails, you pick a coin and someone else in class to throw it [at least 5 yards], would you be willing to take this bet? If not, how much would I have to pay you?

Q6: When is risk neutrality a good assumption?

Q7: Why do people climb mountains, drive motorcyles, play the lottery?

Default ("Credit")

Most loans have credit risk, in that the borrower can default.

Q8: Can the U.S. government default? Do Treasury securities have any default (credit) risk?

Henceforth, assume that a government bond costing $200 promises a 5% interest rate, i.e., $210.

- Assume you are risk-neutral.
- I want to borrow $200 from you. I promise to repay $210.
- However, I may go bankrupt in 1 out of 100 cases, in which case I can repay only $50.

Q9: What is your promised rate of return on my personal bond?

Q10: Do I promise to give you the same rate of return as the Treasury?

Q11: What would you expect my personal bond to return?

Q12: If you extend this loan to me, what rate of return would you expect my bond to give you?

Q13: Would you prefer to make this loan or to put your money into the 5% government bond?

Q14: How much money would you give me in exchange for my promise to pay you $210?

Q15: If the WSJ were to print my bond's interest rate, what interest rate would it print?

Q16: Are (WSJ) quoted interest rates on risky bonds expected rates?

Q17: In the real world, would this interest rate really be high enough?

Default (Credit) and Risk Premia

IMPORTANT:

- **The default premium is compensation to make you break even. It is required to get you to participate even if you are risk-neutral.**

  If you repeat the investment infinitely many times, the average default payment is 0. You get a positive amount if everything goes well, and a negative amount if default occurs. That is, you come out even.

- **The risk premium is extra compensation that gets you above the time premium, and it is only required to get you to participate if you are risk-averse.**

  If you repeat the investment infinitely many times, the risk premium will allow you to earn more than an investor holding Treasuries will earn.

Never confuse the credit premium and the risk premium.

In our world, with a Treasury rate of 5.00% and a quoted bond of 5.81%, there was still no risk premium.

- **Warning:** You must be clear about the distinction between default premia and risk premia. Make sure you know what they are about, and know the difference between these concepts!

Important Generalization

In a risk-neutral world:

IMPORTANT:

Quoted (=Promised) Interest Rate $\geq$ Expected Interest Rate.

Quoted Interest Rate =

Time Premium + Default Premium

Expected Interest Rate =

Time Premium

(We are ignoring other premia for now, such as risk premia, liquidity premia, tax premia, contract premia, etc.)

In our case, the promised interest rate was _____, the time premium is _____, the default premium was _____, and the risk premium is _____.

Risk premia for "fairly safe" bonds from "large, safe" companies are not too high; it is the default premia that jacks up the quoted interest rates.

- IRR computed from promised cash flows is a promised IRR. It is what the WSJ prints.
- The promised IRR is never used in the IRR Capital Budgeting Rule. For capital budgeting purposes, you must use an IRR computed from expected cash flows, not from promised cash flows.

Default (Credit) in Net Present Values

Important: In PV applications, you have to use

$$\frac{\mathcal{E}(\text{Cash Flow})}{1+\mathcal{E}(\text{Discount Rate})}$$

In the real world, the confusion about default rates and risk rates is often worse in PV applications. You must use expected values in both the numerator and the denominator.

- The expected payoff is in the numerator, and it takes care of the default risk of our project. The PV of our loan promising \$210 (i.e., 5.81%),

$$\text{PV} = \frac{\mathcal{E}(\text{Cash Flow})}{1+\mathcal{E}(\text{Discount Rate})} = \frac{\$208.40}{1+5\%} \approx \$198.47$$

- The expected discount rate—*not the promised rate of return*—is in the denominator. It is the opportunity cost of capital on other projects, quoted in terms of their expected rates of return, not in terms of their promised rates of return.
- This denominator (later obtained from the so-called CAPM) does *not* adjust for our project's default risk.
- In a risk-neutral world, every project has the same denominator, regardless of how likely the project or bond is to pay what it promises.

Credit Ratings

- Large corporations have credit ratings, too, ranging from AAA (best) to F.
  - Typical AAA firm has a $\approx 0\%$ probability of default over 10 years.
  - Typical B firm has a 20% probability of one non-payment over 5 years.
  - Typical C firm has a 50% probability of one non-payment over 6–8 years.
- Most of the yield spread of corporate bonds is due to the chance of default (i.e., the credit spread). For example, a Boston Celtics = 9.4%, whereas Treasury = 5.6%.

  The book quotes some sample interest rate differentials and default rates of rated bonds. Although there are other features of these bonds that render the comparison problematic, it gives you an idea of default (credit) worries.

- **Warning:** Never ever confuse expected rates with (higher) promised rates.

  (The 9.4% is not expected!)

  Newspapers and websites virtually never report expected rates.

Credit (Default) Swaps

- Nowadays, you can buy insurance against default. These are called credit (default) swaps.
- Over the counter (OTC). Sellers are often hedge funds who want to speculate on default. Buyers are often mutual funds or pension funds who want to reduce their risk exposure.
- In the event of default, CDS's may pay either a fixed amount or allow the owners of the bond to sell the bond based on a preagreed pricing formula, as negotiated up front.
- In 2006, there was more than $17 trillion worth of credit swaps outstanding.
- This is a rather opaque market—it is possible that the risk of credit is no longer with the holders of the corporate debt.

  Risk is somewhat similar to the housing derivative risk—an obscure bank in Germany may blow up over housing trouble in Kansas.

Preassignment

Important: Reread Chapter 6. Start homework problems.

Big Topic #2 Follows

Uncertainty in Capital Budgeting: PV With Debt and Equity

6-3

This chapter represents several important conceptual leaps:

- Rates of return and NPV under Uncertain Future Cash Flows. (Already Covered.)
- Introduces Payoff Tables and Contingent Claims Valuation.
- Introduces Bonds vs. Levered Equity.
- Introduces Bond Risk vs. Equity Risk.

These are not minor topics, but some of the most important concepts in finance. They are big deals!

Every investment opportunity in our perfect world is fairly priced. You can see yourself as one of two types of investors:

A lender who has provided capital in exchange for the promise of a fixed amount of money. [called leverage]

A levered homeowner (often just called homeowner), who owns the house only with the bundled obligation to repay the loan.

A Financed Project (House/Firm/...)

6-3A

Next Year's Payoffs	Probability	
$100	90%	(Sunshine)
$50	10%	(Hurricane)

The expected rate of return on 1-year Treasuries (and all other 1-year financial instruments) is 5%. The world is risk-neutral. (This is the project example for all the following pages.)

Q18: What would be an appropriate price for this project?

Q19: What is the rate of return on the project in the good state (=promised rate of return)?

Q20: What is the rate of return on the project in the bad state?

Q21: What is the expected rate of return on the project?

Levered Equity (Stock) + Risk-free Bond

You can finance the project either by buying it outright (with $0 mortgage) with financing from your life's savings account, or by buying it with a mortgage and a smaller sum from your life's savings account.
Finance the project with a loan (=bond) promising $50. Markets are perfect, as before. The standard term for the Residual Equity is Levered Equity or Levered Stock or just Stock.

Q22: In the good state, how much do bond and levered equity receive?

Q23: In the bad state, how much do bond and levered equity receive?

Q24: What are the appropriate NPVs on the bond and the levered equity?

Q25: In the good state, how much *rate of return* do bond and levered equity receive?

Project Payoffs	Scheme 1	Scheme 2	
	Firm, FM (=100% Equity)	Bond, DT (Promise=$50)	Levered Equity, EQ
prob(G)=	r=	r=	r=
prob(B)=	r=	r=	r=
$\mathcal{E}$(Payoff) $(= \mathcal{E}(\tilde{C}))$			
$\mathcal{E}$(Rate of Return) $(= \mathcal{E}(\tilde{r}))$			
Discounted Price P_0			
% Financing			

Q26: In the bad state, how much *rate of return* do bond and levered equity receive?

Q27: Draw histograms of the return distributions for all three forms of ownership considered sofar.

Q28: Is full project ownership (=zero leverage) or levered project ownership riskier?

Q29: Is full project ownership (=zero leverage) or bond ownership riskier?

Bond Promising $70

Usually, equity has **limited liability**, which is how we will use it henceforth in the remainder of the course.

- Now price a bond with a promise of $70.
- Enter everything you know.
- Work down the project without financing.
- Work down the pricing of the bond.
- Work back up the pricing of the equity.

Project Payoffs	Scheme 1	Scheme 2	
	Firm, FM (=100% Equity)	Bond, DT (Promise=$70)	Levered Equity, EQ
prob(G)=	$r=$	$r=$	$r=$
prob(B)=	$r=$	$r=$	$r=$
$\mathcal{E}$(Payoff) $(=\mathcal{E}(\tilde{C}))$			
$\mathcal{E}$(Rate of Return) $(=\mathcal{E}(\tilde{r}))$			
Discounted Price P_0			
% Financing			

Borrow $70, Not Promise $70.

We now do not promise $70, but borrow $70. How much do we need to promise to a bank? (Or, if you are the bank, how much do you need as a promise to give money?)

Project Payoffs	Scheme 1	Scheme 2	
	Firm, FM (=100% Equity)	Bond, DT (Promise=?)	Levered Equity, EQ
prob(G)=	$r=$	$r=$	$r=$
prob(B)=	$r=$	$r=$	$r=$
$\mathcal{E}$(Payoff) $(=\mathcal{E}(\tilde{C}))$			
$\mathcal{E}$(Rate of Return) $(=\mathcal{E}(\tilde{r}))$			
Discounted Price P_0			
% Financing			

Leverage and Risk
(and Big Logical Error to Avoid)

Q30: What happens to the riskiness of the stock when more mortgage (say, $70 rather than $1) is taken on?

Q31: What happens to the riskiness of the mortgage when more mortgage (say, $70 rather than $1) is taken on?

(For this project, we can construct increases in mortgage amount where the risk stays the same when the mortgage increases [because the mortgage remains risk-free when the promise increases from $30 to $40]. More generally, in more realistic examples, both claims riskiness go up when the mortgage promised amount increases.)

Q32: What happens to the riskiness of the "firm" (the house overall) when more mortgage is taken on?

A Broader View of Leverage

6-4E

- Leverage = Small movement in lever can create much bigger or smaller movement elsewhere.
- Leverage = The safer part is "outsourced." Small movement in underlying project can make levered ownership much riskier — more upside and more downside.
- Can be done in various ways:
  - With Financial Leverage, as in the example above.
  - With Operational Leverage. Example: Instead of owning safe building and risky technology (together = project medium risky), just lease the safe building. All your money is now in risky technology.

Omitted Sections

6-4F: More than two outcomes.

6-4F+: Recall that you could discount nominal payouts with nominal expected rates of return.

Q33: Can you discount promised payouts with promised rates of return?

Homework Assignment

1. Reread Chapter 6.
2. Read Chapter 7.
3. Hand in all Chapter 6 end-of-chapter problems, due in 7 days.