The Time Value of Money and Capital Budgeting

TVM = Time Value of Money.

IMPORTANT: Reminder:

- **It is important that you keep up with the assignments, or you will (most likely suddenly) realize that you have become lost. Please keep up up with the readings and homeworks to avoid this.**
- **You should have read Chapter 1: Introduction.**
- **You should have read Chapter 2: Time Value of Money.**
- **Do not forget to bring a calculator!**
- **Do not forget to bring the printed class notes!**

Next Two Classes: Also bring WSJ, Section C.

Questions

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- Can you “add” rates of return (or interest rates)?
- How do you work with interest rates?
- What are reasonable measures for interest rates?
- What is a *Basis Point*?
- What does a bank mean if it quotes you an interest rate of 8%?
  Will you end up with $108 in exchange for a $100 investment in one year?
- What is the *Time Value of Money*? What is the *Future Value*? What is the *Present Value*? What is the *Net Present Value*? What is a *Discount Factor*?

Perfect Markets And More

- For the next few chapters, we pretend we live in a so-called *perfect market*. Such a perfect world satisfies four assumptions:

  1. No differences in opinion.

     This assumption does allow for uncertainty, as long as everyone agrees to exactly what it is. This assumption implies no difference in the information set. If there were differences in (relevant) information, investors would come to different opinions.
  2. No taxes.

     Also no government interference and regulation—except costless property rights.
  3. No transaction costs.

     Neither direct nor indirect.
  4. No big sellers/buyers.

     No few investors or firms are special. There are always more where they came from. Of any special investor or firm group, there are infinitely many clones that can buy or sell.

  (Later, you will learn that the perfect market make borrowing and lending rates equal, and allows for a unique price for a good—and what happens if these assumptions are violated.)

- In **this** chapter, we also assume perfect certainty. Thus, you know what the rates of return on every project are and will be.

  (This buys us not having to worry about statistics and attitudes towards risk. This also means there is no difference between rates of returns and interest rates, and implicitly that there are no opinion differences. More on this in four chapters.)

- In **this** chapter, we also assume equal rates of returns per period. (A 1-year bond offers the same annualized [to be explained] rate of return as a 30-year bond.)

  (This buys us not having to worry about the yieldcurve. More on this in two chapters.)

Notation

- Time Convention:
  - 0 = Today, Right Now.
  - 1 = Next period (e.g., day, year, etc.)
  - t = some time period (in the future).
  - T = often to denote a final time period.
- C or C = cash amount. (Often call cash flow, even if it is instant.)
- C_t = instant cash amount at time t.
- $D_{t-1,t}$ = a flow of D (e.g., dividends) over time period $t-1$ to t.
- D_t =, often casual notation for $D_{t-1,t}$.
- $D_{15,20}$ = a flow of D (e.g., dividends) from time 15 through time 20.
- Return vs. Net Return vs. Rate of Return. (Interest rate.)
- r = rate of return.

- This is a bit inconsistent. Dividends are really also paid at one instant in time, and thus should not be subscripted like a flow.
- If the investment is a loan, the rate of return is usually called an interest rate. We will (almost always) use the name "rate of return" and "interest rate" interchangeably.
- Although there is a verbal distinction between a return (C_1), a net return ($C_1 - C_0$), and a (net) rate of return ($C_1/C_0 - 1$), it is rarely explicit. Usually, you are assumed to know what the speaker means.

Rate of Return

IMPORTANT: The rate of return from investing C_0 today and getting C_1 at time 1 is

$$r = r_{0,1} = \frac{C_1 - C_0}{C_0} = \frac{C_1}{C_0} - 1 \quad .$$

This could be called the main formula of finance. With dividends D (or coupons or rent) paid at the *end* of the period (thus not reinvestable to get even more money):

$$r = r_{0,1} = \frac{C_1 + D_{0,1} - C_0}{C_0} = \frac{C_1 + D_{0,1}}{C_0} - 1 \quad .$$

Using aforementioned abbreviations,

$$r = r_{0,1} = \frac{C_1 + D_1 - C_0}{C_0} = \frac{C_1 + D_1}{C_0} - 1 \quad .$$

- The dividend (or coupon or interim payment) yield is $D_{0,1}/C_0$.
- The capital gain is $C_1 - C_0$.
- The percent price change is $(C_1 - C_0)/C_0$.
- The (total) rate of return is the percent price change plus the interim payment yield.

If halfway through the course I casually write $r = P_1/P_0 - 1$ to describe the rate of return, then I am assuming that your P_1 includes any interim payments.

Q1: If the rate of return is positive, can the percent price change be negative?

Q2: You invest $5 and will receive $8 in 10 years. What is your (holding) rate of return?

Q3: Can a rate of return be negative?

Q4: Can an interest rate be negative?

Q5: What is the prevailing interest rate today?

Basis Points and More

Compare 10% to 5%.

- Would you say that 10% is 5% more than 5%?
- Would you say that 10% is 100% more than 5%?

The fact is that it would be easy to misunderstand your meaning.

BASIS POINT: A difference in percent rates times 100.

So, the difference between 5% and 10% is a "500 basis point" difference, and everyone knows what you mean.

Future Value of Money

Q6: You invest $55,000 at an interest rate of 350 basis points above the 5% interest rate. What will you receive at the end of the period?

Q7: If you have $5 and you earn a rate of return of 250%, how much will you have?

Q8: If you have $5 and you earn a rate of return of 40%, how much money will you have?

Q9: What is the formula for the FV of money? How does it relate to the rate of return formula?

Compounding Rates of Return

Q10: If you have $5 and you earn a rate of return of 20% in the first year and a rate of return of 20% the following year, how much money will you have?

he Compounding Formula!

Q12: If the 1-year interest rate is 20% this year, how much money will you get for a $500 investment today in one year? If the following 1-year interest will be 50%, how much will you have after 2 years?

Q13: What is your total holding rate of return? Is it 70%?

Q14: What is the formula for the total holding rate of return, given the two individual rates of return?

bThe Compounding Formula:

$$r_{0,x} = (1 + r_{0,1}) \cdot (1 + r_{1,2}) ... \cdot (1 + r_{x-1,x}) \qquad .$$

If the interest rate remains constant, $r_{t,t+1} = r$ for all T, then

$$r_{0,T} = (1 + r_{0,1})^T \qquad .$$

Q15: If your bank pays you 50% per year, what is your rate of return after 2 years?

Q16: You have \$100. You invest half each in two firms. Firm 1 makes 10% this month. Firm 2 makes 20% this month. How much did your portfolio make in total? Hint: $1.2 \cdot 1.1 = 1.31$.

The difference between adding rates ($r_{0,1} + r_{1,2}$) and compounding returns ($(1 + r_{0,1}) \times (1 + r_{1,2}) - 1$) is the term $r_{0,1} \times r_{1,2}$, which is the interest on the interest. This is also sometimes called the cross-term.

Compounding:
Short Periods to Long Periods.

Q17: The 1 month interest rate is 1%. What is the 1 year rate?

So, after 1 year in the bank, for each \$100 invested, you will have , not !

So, after one week, your weekly interest is $r_{\text{weekly}} \approx 0.1400084...\%$ Note how close $7 \cdot 0.0002$ is to the correct answer!

ncompounding:
Long Periods to Short Periods!

Q19: How good an approximation is simply adding interest? (covered again below.)

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ALGEBRA REFRESHER:

$$x^a = b \iff x = b^{1/a}$$

$$a^x = b \iff x = \frac{\log b}{\log a}$$

Q20: A project for $200 promises to return 8% per year. How much will you have after one month?

he monthly interest rate is 1.5%. There are 30.4 days in the average month. What is the weekly rate?!

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Q22: You are doubling your money in 12 years. What is your rate of return per year?

Q23: A project promises to return 8% per year. How long will it take for you to double your money?

Approximations

IMPORTANT: **Interest rates over time cannot be added, but must be compounded, because you earn interest on the interest! You must always know how and when to compound!**

Q24: Is compounding more like "adding" or "averaging"?

RULE OF THUMB: If both the interest rate and the number of time periods is small,

$$(1 + r_n)^t \approx 1 + t \cdot r_n$$

IMPORTANT: **Adding up instead of compounding gets to be a worse approximation if time increases and if the interest rate increases. (It also matters how much money is**

at stake.)

Jargon

In principle, interest rates (and quotes) are not difficult—but they are tedious and often confusing, because everyone computes and quotes them slightly differently. Sometimes, it is obvious what people mean, sometimes interest rates are intentionally obscure in order to deceive you. You should know what you are talking about. Ask if you are unclear! There can be a lot of money at stake! Arbitrage desks on WS make most of their money on spreads below 20 basis points!

Q25: A bank quotes you 8% interest per year. If you invest $1 million in the bank, what will you end up with?

Quotes vs. Rates: Banks

Unfortunately, many institutions give you interest "quotes," rather than interest rates, and the two are easy to confuse. This is especially bad with annualized interest quotes. There are many "pseudo interest rates" which are really "interest quotes," not "interest rates."

IMPORTANT: Banks and Lenders typically calculate and pay daily interest rates, though they only credit to accounts once per month. Banks' daily interest rate calculation is the quoted annual interest rate divided by 365 ($r_d = r_y/365$). (Note: some banks use 360 days.)

Some banks quote

Interest rate: 8% compounded daily. Effective annual yield: 8.33%.

Q26: Is 8% an annual interest rate?

Q27: Is the "effective annual yield" an interest rate?

PS: Banks often name plain interest quote for loans (e.g., mortgages), and EAR for CDs and savings. Caveat Emptor: Know what you are getting!

Quotes vs. Rates: Government Bonds.

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At a Treasury auction, the government sells Treasury bills that pay \$10,000 in 180 days. If the government discount quote is 10% (which is absolutely *not* an interest rate), then it means you can purchase the Treasury bill at the auction for \$9,500, because they use the formula

$$\begin{aligned} \text{TB Price} &= \$100 \cdot [100 - (\text{days to maturity}/360) \cdot \text{discount quote}] \\ &= \$100 \cdot [100 - (180/360) \cdot \mathbf{10}] = \$9,500 \end{aligned}$$

Do not bother remembering this formula. This is hairy stuff—not conceptually, but detail-wise. If you are not going into to become a bond trader, you just need to know that it exists. I do not remember this formula, either. I looked it up. It may even be wrong.

Financial newspapers (e.g., WSJ) print "95" instead of "9,500," because it is shorter, so T-bills are quoted in units of 100.

Q28: Assume the quote is indeed 95. If you invest \$1, how much will you receive in 6 months?

Q29: Is the 10% discount quote an interest rate or an interest quote?

Present Value and Capital Budgeting

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IMPORTANT: The present value of cash C_t at time t is

$$\mathsf{PV} = C_0 = \frac{C_t}{1+r_{0,t}} \quad .$$

- **The quantity $1/(1 + r_{0,t})$ is called the discount factor. It is the factor that is multiplied to a future cash flow in order to obtain the future cash flow's current value.**
- **The quantity $r_{0,t}$ is called the discount rate, because it is the interest rate that is used to obtain the discount factor.**
- **In this context, the discount rate is also often called the (opportunity) cost of capital, because you should think of it either representing your alternative investment opportunities (if you have money) or your cost of borrowing (if you need money).**

  In our perfect market, the two are the same. That is, in our financial markets, you can invest into infinitely many alternatives for a rate of return that is exactly to your cost of borrowing.

Q32: How does the price of a bond change if the economy-wide interest rate changes?

Net Present Value

Q33: If you will receive $7 next year and another $7 in two years, and the prevailing (alternative) interest rate [or cost of capital] is 40%, do you have the equivalent of $14?

Q34: What do you value this project as of today?

Q35: What is the formula?

Q36: If this project costs $8, should you take this project?

Q37: **If the cost of capital were 80%/year, should you take this project?**

NPV Formula

IMPORTANT: The net present value is

$$\text{NPV} = C_0 + \frac{C_1}{1+r_{0,1}} + \frac{C_2}{1+r_{0,2}} + ...$$

$$= \sum_{t=0}^{\infty} \frac{C_t}{1+r_{0,t}} \quad . \; It is called "net," because the first cash flow$$

C_0 is often negative.

Logical Foundation:

Here is how a perfect world w/o uncertainty must work:

- **The NPV rule is optimal (other rules leave money on the table),**
- **and positive NPV projects must be scarce,**

The proof is almost trivial. For example, presume that, in our perfect market, you can borrow or lend m 8% anywhere today. The NPV formula says you will not make money on projects that cost $1 today ar $1.08 next year. It says you should take all projects that yield more than $1.08 next year. Now, presume t have (infinitely) many investment opportunities that cost $0.99 and yield $1.08. (The NPV is positive.)

Q38: **How do you get rich?**

If such projects are in limited supply, you (and everyone else) would buy up all such projects, until the p equilibrium price has increased to make the project zero NPV.
(If you can short projects, and you have willing buyers for negative NPV projects, you can just sell th thereby invert the argument.)

Growth as Investment Criterion

Q39: Are Fast-Growing Firms Better Investments than Slow-Growing Firms?

Assume the prevailing interest rate (cost of capital) is 20%/year.

Company *FastLane*: $100 today, $200 next year.

Company *DeadBoring*: $100 today, $50 next year.

Q40: What is the price of *FastLane* today?

Q41: What is the price of *DeadBoring* today?

If you put money purchasing shares of these firms, for each dollar invested today, by next year:

Q42: What is your expected rate of return in *FastLane*? On *DeadBoring*? Which is the better deal?

Q43: What would it take for *FastLane* to be better than *DeadBoring*? (How critical are our perfect market assumptions, such as no taxes?)

Homework Assignment

1. **Reread Chapter 2.**
2. **Read Chapter 3.**
3. **Hand in all Chapter 2 end-of-chapter problems, due in 7 days.**