CHAPTER 14[§]

The CAPM: The Theory and its Limits

Where does the CAPM equation come from?

This chapter appears in the Survey text only.

THIS chapter explains the CAPM theory in detail *and* its shortcomings. Actually, because we have already discussed all the necessary ingredients—mostly in Chapter 12 which is absolutely necessary background for this chapter—this chapter is surprisingly simple. Really.

14·1 The Theory

14·1.A. The Logic and Formula

Here are the underlying mathematical facts, which were already explained.

We have already covered every necessary aspect of the CAPM theory in earlier chapters:

- In Section 8·3.D, we learned that the portfolio held by investors in the aggregate is the (market-capitalization) value-weighted portfolio.
- In Section 12·3, we learned that the combination of Mean-Variance Efficient portfolios is itself MVE.
- In Section 12·5, we learned that each stock i in a MVE-portfolio must offer a fair expected rate of return for its risk contribution to E. Formula 12.11 states that for each and every security i in the MVE portfolio E, this means that

$$\mathcal{E}(\tilde{r}_E) \;=\; r_F \;+\; [\mathcal{E}(\tilde{r}_E) - r_F] \cdot \beta_{i,E}$$

Here is the CAPM Theory.

The CAPM is a theory that has only one hypothesis and only one implication.

CAPM Hypothesis: The CAPM hypothesis is that each and every investor chooses an MVE portfolio.

Investors may not hold MVE portfolios.

Unlike the method for choosing weights for an MVE portfolio—which are just mathematical optimization techniques and which have to be correct—the CAPM is a real theory that may not hold in practice: investors might have different investment objectives and therefore not hold MVE portfolios.

CAPM Implication: If the CAPM hypothesis is correct, then the mathematical implication is that the aggregate value-weighted market portfolio is also MVE.

The CAPM could be wrong in practice.

If the CAPM hypothesis is not correct, there is no particular reason to expect the value-weighted portfolio to lie on the MVE Frontier. Instead, the value-weighted market portfolio could have risk-reward characteristics that place it far inside the MVE Frontier.

This is it: the Capital-Asset Pricing Model, which won the 1990 Nobel Prize in Economics.

IMPORTANT: The CAPM conjectures that all investors purchase MVE portfolios. As a necessary mathematical consequence, the value-weighted market portfolio is also MVE.

OK, let's do with algebra and logic.

Put altogether again, the logic of the CAPM is:

Mathematical Fact: To enter an MVE Frontier portfolio E, each stock in E has to offer an appropriate reward for its risk. This formula, which relates its expected rate of return to its covariation with E, is

$$\mathcal{E}(\tilde{r}_i) \;=\; r_F + [\mathcal{E}(\tilde{r}_E) - r_F] \cdot \beta_{i,E}$$

Stocks that offer expected rates of return higher than suggested by this formula would generate too much aggregate investor demand; stocks that offer lower expected rates of return would generate too little aggregate investor demand.

CAPM Theory Assumption: Every investor purchases an MVE portfolio.

$\Longrightarrow$ **Implication:** The value-weighted market portfolio is MVE.

$\Longrightarrow$ **Implication:** that

$$\mathcal{E}(\tilde{r}_i) \;=\; r_F + [\mathcal{E}(\tilde{r}_M) - r_F] \cdot \beta_i$$

In this case, the market portfolio consisting only of risky securities must be the tangency portfolio T.

DIG DEEPER

This is the "Sharpe-Lintner" CAPM. There is also a "Fisher Black" CAPM, in which there is no risk-free rate of return. Everything works the same, except that the risk-free rate is replaced by the constant a. All stocks in the economy follow the CAPM formula

$$\mathcal{E}(\tilde{r}_i) = a + [\mathcal{E}(\tilde{r}_M) - a] \cdot \beta_i \quad (14.1)$$

with respect to the value-weighted market portfolio M.

14·1.B. Some Odds and Ends

➤ The theoretically correct market portfolio in the CAPM is the market-capitalization value-weighted portfolio, consisting of all possible investment assets in the economy, not just stocks in the U.S. stock market.

The market portfolio is theoretically clear, but practically difficult to identify.

However, in traditional investments use, the assumed goal is to pick the best portfolio among publicly traded stocks. It is as if the CAPM theory was restated to conjecture that investors seek to optimize the risk/reward relationship only within their portfolios of publicly traded domestic stocks. In this case, the proper market portfolio would be the market-capitalization value-weighted U.S. stock market portfolio. However, even this portfolio is difficult to obtain every day. Fortunately, the value-weighted U.S. stock market portfolio has high correlation with other broad stock market indexes, such as the S&P500. Thus, the S&P500 is often used as a reasonably good substitute.

Although this is reasonable use for CAPM testing and corporate CAPM application, it is *not* good investment advice: you should definitely diversify across more assets than just domestic stocks. International stocks, commodities, real-estate, and even education represent other investment classes that are readily available to join smart investors' portfolios.

In the real world, different investors have different investment opportunities. For example, you can invest in your house or you can invest your education and reap the rewards—but I cannot invest in *your* house and *your* education. The CAPM ignores this issue altogether. But if you have too much invested in some untraded assets (education or your specific house), you might not want the same kind of stocks to diversify your educational investment risk away. That is, you might not want stocks to minimize your portfolio variance only. Instead, you might want to purchase relatively more stocks that go up when your other untraded assets go down, and vice-versa.

➤ Transaction costs would be high enough to prevent ordinary small investors from themselves purchasing the widely diversified portfolio prescribed by the CAPM. Mutual Funds (Section 8·3.B), however, have made this relatively easy.

Mutual Funds facilitate holding the market portfolio.

➤ What if the market portfolio does not sit on the MVE Frontier? Then there will be stocks for which the CAPM Formula 14.1 does not hold. And then it should not be assumed that the CAPM formula is the appropriate relation predicting stocks' expected rates of return.

If the market portfolio is not MVE, then the CAPM does not hold and the CAPM formula cannot be used.

Indeed, then anything else could conceivably explain higher or lower required rates of return. For example, instead of firms with higher market betas, it may be firms with higher P/E ratios or firms with older CEOs or firms containing the letter "Z" that may end up having to offer higher expected rates of return in order to induce investors to willingly purchase and hold them in equilibrium. Naturally, such a situation would make it very difficult for corporations to determine the appropriate cost of capital that their projects should offer.

Higher beta implies higher expected rate of return.

- Because $\mathcal{E}(\tilde{r}_M) > r_F$, the intuition of the CAPM formula is that stocks with higher stock-market betas have to offer higher expected rates of return. Stocks with high betas are less helpful in reducing the risk of an investor who already holds the market portfolio.

From the perspective of a widely diversified investor, a tiny bit of zero-beta stock is just as good as investment in the risk-free security.

- A risky project that has a beta of zero need only offer an expected rate of return no higher than the risk-free rate itself. This is because each investor would only hold a tiny amount of this security. Having one cent in the risk-free rate or in a security whose return is expected to be the same and with no correlation to the rest of the portfolio is practically the same. In the real and naturally not perfectly CAPM world, this is not exactly correct—but it is often still a reasonably good approximation.

Solve Now!

Q 14.1 What is the main CAPM hypothesis, and what is the main CAPM implication?

Q 14.2 Write down the CAPM formula without looking at the text. You must memorize it!

Q 14.3 Under what circumstances is the market-beta a good measure of risk?

Q 14.4 What can plotting both the stock-market portfolio and the risk-free rate into the MVE Frontier graph tell you about the model and about the security markets line?

Q 14.5 What can checking whether every single security follows the CAPM formula (i.e., that every single possible investment is on the security markets line in a graph of expected rate of return against beta) tell you about the MVE Frontier (a graph of expected rate of return against standard deviation)?

Q 14.6 If all but one security are right on the security-markets line, is the market portfolio MVE?

Q 14.7 Is the CAPM market portfolio the value-weighted or the equal-weighted portfolio?

Q 14.8 Can the S&P500 be used as a proxy for the market?

Q 14.9 In the formula, why do higher beta stocks offer higher expected rates of return?

Q 14.10 If there is a risk-free security, does the stock market portfolio have to be the tangency portfolio for the CAPM to hold, or can it be any portfolio on the mean-variance frontier of risky assets?

Q 14.11 Is it possible for a risky stock to offer an expected rate of return that is less than the risk-free rate?

14·2 Does the CAPM Hold?

14·2.A. Listing All the CAPM Assumptions

Let's judge the CAPM Model Assumptions.

We have sneaked in the CAPM assumptions one at a time, perhaps to make them appear more palatable. It is excusable if they have slipped your mind by now. To help you judge how likely it is that the CAPM assumptions are reasonably satisfied in the real world, it is worthwhile to repeat them all at once. These are all the assumptions that we have used to conjecture that each and every investor buys an MVE portfolio:

Market Assumptions:

- The market is perfectly competitive.
- There are no transaction costs.
- There are no taxes.
- The investment opportunity choices are identical for each investor. Therefore, the value-weighted market portfolio is identical for all investors. It consists of all assets that can be invested in. It includes such assets as real estate, bonds, international markets, etc.
- The previous assumption means that are no untraded assets that only some, but not other investors can hold. That is, you cannot have your own house, or your own children, or your own education, or your own labor income, or your own executive stock options.

Informational Assumptions:
- ➤ Financial markets *are* efficient: as we will discuss in Chapter 16, this means that the market does not ignore information in the setting of financial prices, which an investor could use to outperform the market securities line. You cannot use public information to pick stocks better than the market can.
- ➤ There is no *actual* inside information that allows some investors to pick stocks better than the market can.
- ➤ There is no *perceived* inside information. That is, investors do not *believe* that they are able to pick stocks better than the market can.
- ➤ All investors share the same "opinion" about security expected rates of return and security variance/covariances.
- ➤ All model parameters are perfectly known:
  - The expected rate of return on each and every stock (including the market portfolio) is known.
  - All covariances (and thus betas) are known.
  - If you as a researcher want to test the CAPM, i.e., whether other variables matter, you must also know possible alternative return factors (e.g., firm size) that investors can use.
- ➤ There are no managerial agency problems (i.e., managers enriching themselves) that can be changed if a large investor holds more of the particular stock with agency problems.

Preference Assumptions:
- ➤ Investors care only about their portfolio performance. They do not care about other characteristics of their holdings (e.g., socially responsible corporate behavior).
- ➤ Investors like mean and dislike variance. This fully describes their portfolio preferences. (It does assume that investors do not care, e.g., about skewness of their portfolio's future investment rate of return.

↦ *Skewness 11·1 on Page 240*

- ➤ Agents maximize their portfolio performance independently each period. (Although this may appear innocuous, it implies that investors do not choose their portfolios to insure themselves against changes in future investment opportunities. You cannot buy stocks that you believe to pay off more in a recession, because you believe then will be a good time to double up investment.)
- ➤ Investors do not care about how their portfolios perform relative to other variables, including variables related to their own personal characteristics (except for risk-tolerance). For example, investors must not care about earning a different rate of return in states in which they find themselves ill or states in which inflation is high.

The CAPM is just a helpful model, not perfect reality.

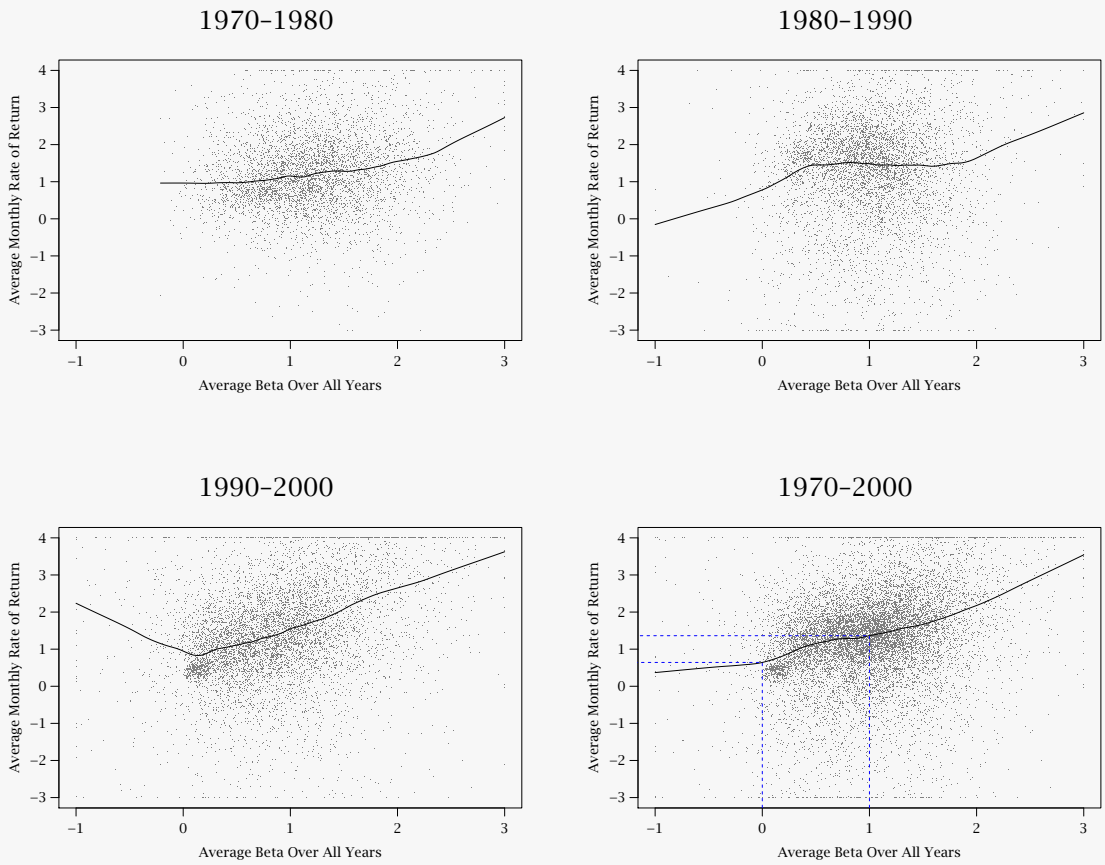
If these assumptions hold, the CAPM will hold. Of course, it could also be that the value-weighted market portfolio just happens to lie on the MVE frontier, in which case the CAPM security markets line would just happen to be a line by accident. Fortunately, the market need not lie *perfectly* on the MVE frontier, nor must the assumptions hold *perfectly* in order for the CAPM to be a useful model. After all, the CAPM is *only* a model, not reality. Only mathematics works the same in theory and in practice—economic models do not. The real question is whether the CAPM assumptions are sufficiently badly violated to render the CAPM a useless model—and in what context.

14·2.B. Is the CAPM a good representation of reality?

You do not hold the stock market portfolio, therefore the CAPM is incorrect. True, but...

In real life, we know that some investors purchase portfolios other than MVE portfolios. This should not come as a big surprise: chances are that you yourself are one of these investors, even if you are only a small investor. Therefore, in the strictest sense, we already know that the CAPM does not hold. But, again, the CAPM is only a model. The question is whether the CAPM simplification of reality is helpful in understanding the world, i.e., whether the model is sufficiently close to reality to be useful. Perhaps most of the real big investors "who matter" do invest in portfolios sufficiently similar to MVE portfolios, so that the value-weighted market portfolio is on the MVE Frontier, meaning that the CAPM "roughly" works.

Figure 14.1: Average Historical Rates of Return Against Historical Market Beta.



The web chapter on empirical asset pricing examines, among other issues, noise and non-stationarity problems when testing investments models.

The empirical relationship is graphed.

Unfortunately, the empirical evidence suggests that the market portfolio does not lie too closely on the MVE Frontier, that the security markets line is not perfectly linear, and that expected security returns relate to other characteristics in addition to beta. Figure 14.1 plots average monthly returns against stock betas. Care must be exercised when viewing these figures, because there are few stocks with betas below 0 and above 2. Therefore, the smoothed lines are not too reliable beyond this range. In the 1970s, the relationship between beta and average (monthly) return was generally upward-sloping, though not perfectly linear. In the eighties, although stocks with betas below 0.5 and 1.5 had similar average rates of return, stocks outside the 0.5 to 1.5 beta range (not too many!) had a clear positive relationship. In the 1990s, firms with positive betas showed a nice positive relationship, though the (few) stocks below $\beta = 0$ did not.

The most interesting plot is the overall graph, plotting the relationships from 1970 to 2000. The typical stock with a beta of 0 earned a rate of return of about 8% per annum, while the typical stock with a beta of 1 (i.e., like the market) earned a rate of return of about 18% per annum. Not drawn in the figure, the average stock with a beta of 2 earned about 217 basis points per month (30% per annum), and the average stock with a beta of 3 earned about 354 basis points per month (50% per annum). These 30 years were an amazingly good period for financial investments! The figure shows also how there was tremendous variability in the investment performance of stocks. More importantly from the perspective of the CAPM, the relationship between average rate of return and beta was not exactly linear, as the CAPM suggests, but it was not far off. If we stopped now, we would conclude that the CAPM was a pretty good model.

But look back at Figure 13.5. The empirical evidence is not against the CAPM in the sense of the first three plots (linearity)—it is against the CAPM in the sense of the last three plots (better alternative classifications). Although we cannot see this in Figure 14.1, the CAPM fails when we split these stocks into groups based on different characteristics—and academics seem to come up with about five new different characteristics per year. The empirical reality is somewhat closer to the latter three figures than it is to the idealized CAPM world. This implies that market beta seems to matter *if* we do not control for certain other firm characteristics. For example, some evidence suggests that firms that are classified as "small growth firms" by some metrics generally underperform "large value firms"—but neither do we really know why, nor do we know what we should recommend a corporate manager should do about this fact. Maybe managers should pretend that they are growth firms—because investors like this so much they are willing to throw money at growth firms—but then act like value firms and thereby earn higher returns?! But we still have another "little problem" (irony warning)—we finance academics are not exactly sure what these characteristics are, and why they matter.

But this is deceptive—the CAPM fails against specific better alternatives.

14·2.C. Professorial Opinions on the CAPM

Different academics draw different conclusions from this evidence—and it should leave you with a more subtle perspective than we academics would like. See, we professors would be happy if we could tell you that the CAPM is exactly how the world works, and then close the chapter. We would probably even be happy if we told you that the CAPM is definitely *not* the right model to use, and then close the chapter. But it is not so simple. Yes, some professors recommend outright against using the CAPM, but most professors still recommend "use with caution." Now, before I give you my own view, be aware that this is just my own assessment, not a universal scientific truth. In any case, you positively need to be aware of the advantages and disadvantages of the CAPM, and know when to use it and when not to use it.

You need to know pros and cons.

My own personal opinion is that although the CAPM is likely not to be really true, market beta is still a useful cost-of-capital measure for a corporate finance manager. Why so? Look at the last three plots in Figure 13.5 again: If you have a beta of around 1.5, you are more than likely a growth firm with an expected rate of return of 10% to 15%; if you have a beta of around 0, you are more than likely a value firm with an expected rate of return of 3% to 7%. Thus, beta would still provide you with a decent cost of capital estimate, even though it was not beta itself that mattered, but whether the firm was a growth or a value firm. (Market beta helped in indicating to you whether the firm was a growth or a value firm in the first place.) Admittedly, using an incorrect model is not an ideal situation, but the cost-of-capital errors are often reasonable enough that corporate managers generally can live with them. And the fact is, if corporations cannot live with these errors, we really do not know what to recommend as a better alternative to the CAPM!

My personal opinion.

In sum, although the CAPM formula is mistaken, it continues to be the dominant model for the following reasons:

Why the CAPM continues to be and should be used.

1. Although the market portfolio does not seem to lie directly on the MVE Frontier, it does not appear to be *too* far away.
2. The CAPM provides good intuition for the characteristics that should determine the expected rates of return that have to be offered by stocks: stocks that help diversified investors achieve lower risk should be sellable even if they offer lower expected rates of return.
3. For many purposes and many stocks, the CAPM provides reasonable expected rates of return, not too far off from those gleaned from more complex, difficult, and/or arbitrary models.
4. There is no good alternative model that is either simpler to use or convincingly better in providing appropriate expected rates of return for investments.
5. The CAPM is *the* gold standard for cost-of-capital estimations in the real world. It is in wide use. Understanding it (and its shortcomings) is therefore crucial for any student with an interest in finance.

In contrast, my advice to an investor would be *not* to use the CAPM for investment portfolio choices. There are better investment strategies than just investing in the market, although wide diversification needs to be an important part of *any* good investment strategy.

Another interesting question is *why* the CAPM fails. In my own opinion, most investors do not invest scientifically. They buy stocks that they believe to be undervalued and sell stocks that they believe to be overvalued. Why do even small and relatively unsophisticated investors—often relying on investment advice that qualifies as the modern equivalent of hocus-pocus—believe they are smarter than the financial markets, whilst even professional investors have difficulties beating the market? This is perhaps the biggest puzzle in finance.

Although people's tendencies to categorize choices might have given us some hope that people at least maximize MVE within the domain of their own pure market investment portfolios, perhaps subject to their idiosyncratic estimates of expected rates of return, the evidence suggest that investors are not even internally consistent. Even among the stocks for which they hold opinions about high or low expected rates of return, investors tend not to choose securities to reach their own best MVE Frontiers.

Unfortunately, if every investor behaves differently, it is also not clear what the expected rate of return on a particular investment really should be or has to be—other than that it need not necessarily be that suggested by the CAPM. It could be high, it could be low, it could be anything.

This phenomenon is the domain of behavioral finance, eventually to be further expounded in a web chapter.

14·2.D. Why not Optimization instead of the CAPM?

What are our alternatives to the CAPM? Investors have good, although tedious alternatives...

Why could we not just rely on optimal portfolio theory (Chapter 12) instead of on the CAPM? From the perspective of an investor, this is a feasible and probably better choice. The only drawback is that instead of believing that the market portfolio is MVE, i.e., that a part of the investor's portfolio should be allocated to it, the investor has to solve the more general problem of where the MVE Frontier is. This is tedious, but possible. Even so, in many situations, i.e., when not too much money is at stake, even though the stock market portfolio is not exactly MVE, it is probably still close enough for most investors to just choose it anyway in order to avoid the pain of computation. Nevertheless, the CAPM should not be used for investment purposes where more than $100 million is at stake. There may be better investment strategies. Again, for smaller investment portfolios, the advice of purchasing a combination of the value-weighted market portfolio (including international stocks and other assets) and a risk-free rate is probably reasonably close to an MVE portfolio.

...while corporations really have few or no alternatives.

From the perspective of a firm, however, relying on optimal portfolio theory is not a feasible choice. Corporate managers need to be able to compute what expected rate of return investors would demand for a particular project. If they do not know what investors want in equilibrium, managers are in a guessing game. Take the CAPM for what it is useful for: it is a model that provides decent guesstimates of appropriate cost of capital for non-financial projects, in which the exact discount rate is not too critical. Do not use it if a lot of money can be lost if the discount rate is misestimated by a percent or two.

Solve Now!

Q 14.12 List as many assumptions about the CAPM that you can recall. Which ones are most problematic?

14·3 Portfolio Benchmarking with the CAPM?

Can the CAPM Formula be beaten in a CAPM world?

The performance evaluation of investment managers is an example of how problematic the application of the CAPM formula can be. The typical CAPM use in **portfolio benchmarking** has investors compute the beta of portfolios held by their investment managers, and check if the manager's portfolio return over the year beat the securities market line. Unfortunately, if the CAPM holds, the security markets line cannot be beaten. After all, the definition of the MVE Frontier is that it is the set of portfolios that offers the best possible combination of risk and return. If the CAPM does not hold, then the security markets line has no reason for living. It would be totally ad-hoc. Strictly speaking, the CAPM cannot be used for portfolio benchmarking.

There is a good intuitive reason to use the CAPM, anyway.

Nevertheless, the CAPM formula gives good intuition on what constitutes good performance contribution to a widely diversified investor. It also works if the fund manager with the presumably better information and stock picking ability is too small to move prices. This explains why the use of the CAPM formula for performance evaluation continues—despite all its problems.

14·4 Summary

The chapter covered the following major points:

- The logic of the CAPM is that if every investor holds a mean-variance efficient portfolio, then the (value-weighted) market portfolio is mean-variance efficient. This in turn means that stocks follow the linear relationship between the expected rate of reward and market-beta (the security markets line).
- The CAPM relies on many assumptions. It is only a model.
- The CAPM is a useful simplification in certain contexts (such as capital budgeting), because it is intuitive, easy to use, and often gives a reasonable enough cost-of-capital estimate.
- The CAPM is not a reasonable description of reality in many other contexts. It should not be used for stock investing purposes.
- There are no good alternatives to the CAPM. This is why the CAPM, with all its faults, is still the predominant model in most corporate contexts.

14·5 Theory: CAPM Alternatives!?

In 2007, about 75% of all finance professors recommended the CAPM for use in a corporate capital budgeting context. About 5% recommended using the so-called APT, and 10% recommended using the so-called Fama-French factors. Not surprisingly, these two alternative models have advantages and disadvantages relative to the CAPM. It is impossible to explain them fully in a corporate finance course, but I want to give you at least a sketch of them, anyway.

14·5.A. The Arbitrage Pricing Theory (APT) and Intertemporal CAPM (ICAPM)

For an analogy, recap the CAPM.

The first alternative is an extension of the ordinary CAPM, called the ICAPM. The second is called the **Arbitrage Pricing Theory** (**APT**). In practical use, the two are indistinguishable, so I will just treat them as one and the same model here. Let's think back as to how the CAPM works:

1. The CAPM asks you to measure how each stock's rate of return moves together with the overall stock market rate of return. This is its market-beta.
2. The model's intuition is that investors dislike stocks that move together with the stock market and like stocks that move against the stock market.
3. The CAPM tells you the exact formula by which you should receive a higher average rate of return for firms that expose you to a lot of covariation with the stock market. It may be

$$\mathcal{E}(\tilde{r}_i) \;=\; 4\% \;+\; 5\% \cdot \beta_{i,\mathrm{M}}$$

where the second subscript reminds you that this beta measures a stock's sensitivity with respect to the market.

You might care about factors other than the rate of return on the stock market.

Now let's assume that stocks differ not only in how they move with or against the stock market, but also in how they move with or against other economic factors, say, the oil price. You might care about oil price changes because your business may do poorly if energy costs rise. Therefore, if you can find a stock that increases in value when oil prices rises, you would consider this stock as good insurance against bad business—just as you consider a stock that goes up when the market goes down as good insurance against market downturns in the CAPM framework. (If you are in this situation, chances are that you would really like to hold stocks like Exxon or Chevron.)

Different stocks can have different exposures to these other economy-wide factors.

How can you measure whether a stock goes up or down with the oil price? Simple—you get this measure the same way that you get a measure of whether a stock goes up or down with the stock market. For each stock, you run a time-series regression, in which the independent variable is not the rate of return on the stock market but the oil price change,

$$\tilde{r}_i \;=\; a + \beta_{i,\text{Oil Price Change}} \cdot (\text{Oil Price Change})$$

This gives you for each stock a beta that measures how its rate of return moves with oil price changes. A stock that has a very large $\beta_{i,\text{oil price change}}$, say 5, would go up a lot if the oil price increases—think Exxon. A stock that has a negative $\beta_{i,\text{oil price change}}$, say -3, would go down when the oil price increases—think UPS (which has to pay more for gas when the oil price increases).

And here is the analogy.

Would you be willing to pay more for a stock that acts as an insurance against oil price increases? If your livelihood is adversely affected by oil price changes, then the answer is probably yes. The more important question is whether this is also the attitude of most investors in the market. If it is, then stocks like Exxon, which have high $\beta_{i,\text{oil price change}}$'s, would be more desirable. They would not have to offer as high a rate of return as other stocks that have a low $\beta_{i,\text{oil price change}}$. The APT then gives you a formula that relates the oil-price-change beta (and other betas like it) to the expected rate of return on a stock—something like

$$\mathcal{E}(\tilde{r}_i) \;=\; 4\% \;+\; 5\% \cdot \beta_{i,\mathrm{M}} \;-\; 3\% \cdot \beta_{i,\text{Oil Price Change}}$$

You can now use the formula the same way you used the CAPM formula. To recap, the APT works like the CAPM, but allows more than just one beta.

1. The APT asks you to measure for each stock how it moves with respect to factors (like the oil price) that you decide on. This gives you, for each stock, a set of market-betas, one for each factor.
2. The intuition is that investors like stocks that have high or low betas with respect to these factors. (The sign depends on investors' preferences towards the factor.)
3. The APT tells you the exact formula by which you should receive a higher average rate of return for firms that expose you to bad covariation with respect to the factors that matter.

What are the APT Factors?

Common APT models use as factors interest rate changes, GDP changes, bankruptcy risk, the returns of growth stocks, and the returns of small firms. Each stock then has a beta with respect to these factors. And an APT formula relates the average rate of return to these betas.

APT flexibility is both good and bad.

Unfortunately, the APT is even harder to use than the CAPM. The good news is that it allows you to specify that investors care about factors other than the overall stock market. You then use the beta of your project with respect to the market to determine the appropriate expected rate of return. The bad news is that it allows you to specify that investors care about factors other than the overall stock market. The problem is that the APT does not give you any guidance on what these factors should be. What does academia recommend? Sorry, there is no consensus of what the best APT factors are. So the APT's flexibility is both a blessing and a curse.

Canned usage is easy IF you pay for it.

Most commonly, corporations rely on third party vendors that have developed such APT models to at least get a second opinion on their overall cost of capital. (This is rarely done for individual projects, even though we know that this would have to be done project-by-project.) The vendor reports the factors that they like to use (the market beta, and the oil price change in our example) and the "premiums" (4%, 5%, –3% in our example), and estimates your firm's betas with respect to these premiums. You can multiply the factors with the premiums to obtain an alternative measure for the cost of capital. Alas, like the CAPM, there is no guarantee that any one particular APT model is the right model. In fact, two APT vendors can easily derive completely different cost of capital estimates. You have to judge which one is better. In other words, use the APT at your own risk.

Solve Now!

Q 14.13 Explain how the APT model is similar to but more general than the CAPM.

14·5.B. The Fama-French-Momentum (-And-More) Model

The Fama-French Factors plus Momentum.

While the APT developed out of a tradition of theoretical models with empirical applications, another set of models has come out of the tradition of empirical research. The most prominent empirical regularities right now seem to be:

1. **Momentum** Stocks tend to perform better if they have had high stock returns over the previous 12 months, but excepting the most recent month. (Omitting this last month is very important.) Momentum is a very robust predictor.
2. **Value** Stocks tend to perform better if they have high accounting book value of equity (explained in Chapters 18 and 19) divided by the market value of equity. Firms that fit this criterion are called **value firms**, while firms that have higher market values than accounting book values are called **growth firms**. A typical value firm is "boring," like Proctor&Gamble. A typical growth firm is "exciting," like Google or Apple. In the long run, the better stock return performance of value over growth has been a very robust relationship, too—even though there were some periods when it did not hold, first and foremost during the late 1990s.
3. **Size** There is some evidence that smaller firms perform better than larger firms. The role of firm size is not as strong and robust as the two preceding effects.

The latter two regularities are usually called the **Fama-French factors**, because it was Eugene Fama and Ken French who investigated them most thoroughly. Please don't think that these three empirical regularities are the only ones. There are literally dozens more (accounting accruals and net issuing activity are particularly noteworthy), although these three factors are generally considered to be the most important ones known today. For other determinants of good rate of return performance, you really have to read an investments text book.

Use of the Model in a Corporate Context

Finding exposures—like APT exposures.

You can use these three regularities in an APT-like version of this model in a corporate context. Let me try to sketch how this would work. Ken French posts the historical rates of returns for the equity premium and the three factors on his website:

XMKT, the equity premium, which is the average rate of return on the stock market net of the risk-free rate. The average rate of return on XMKT from 1927 to 2006 (their sample period) was about 8.5%.

UMD, the momentum portfolio, which is the average rate of return on firms having done well over the last 12 months minus the average rate of return on firms having done poorly (both omitting 1 month). The average rate of return on this portfolio from 1927 to 2006 was about 8.9%.

HML (high-minus-low), the "value" portfolio, i.e., the average historical rate of return on stocks with high accounting book value relative to market value, minus the opposite. The average rate of return on this portfolio from 1927 to 2006 was about 4.6%.

SMB (small-minus-big), is the "small firm" portfolio, i.e., the average historical rate of return on stocks of small firms minus the equivalent for large firms. The average rate of return on this portfolio from 1927 to 2006 was about 3.8%.

You would first run a time-series regression of your own project's (i's) historical rates of returns *net of the risk-free rate* on the four time-series:

$$\tilde{r}_i - r_F = a_i + b_i \cdot \text{XMKT} + c_i \cdot \text{UMD} + d_i \cdot \text{HML} + e_i \cdot \text{SMB} + \text{noise}$$

Note the abnormal return of 3%, which should not repeat.

Now let's say that you estimated your project's coefficients to be $a = 3\%$, $b = 2$, $c = 0$, $d = 0$, and $e = 0$. Well, then your particular stock behaves almost like a CAPM stock with a market-beta of 2, because your reduced Fama-French model is

$$\mathcal{E}(\tilde{r}_i) - r_F = 3\% + 2 \cdot \mathcal{E}(\text{XMKT}) = 3\% + 2 \cdot [\mathcal{E}(\tilde{r}_M) - r_F] \tag{14.2}$$

Note that the risk-free rate intercept is already on the right-hand side, so your 3% estimated intercept would be an excess rate of return that your stock has earned historically, above and beyond what the Fama-French model would have suggested. You would therefore also not expect this extra 3% rate of return to repeat.

Using one form of this model to predict expected rates of return.

Let's work out the hurdle rate for your project. If you believe the future equity premium to be 5% and the Treasury risk-free rate to be 4%, then you would expect your stock's rate of return to be

$$\mathcal{E}(\tilde{r}_i) - 4\% = 2 \cdot 5\%$$

$$\mathcal{E}(\tilde{r}_i) - r_F = \beta_{i,\text{XMKT}} \cdot \mathcal{E}(\text{XMKT})$$

Your expected rate of return suggested by this model for your project would therefore be $\mathcal{E}(\tilde{r}_i) = 4\% + 2 \cdot 5\% = 14\%$. Note how the Fama-French application omits the 3% from Formula 14.2 here—the reason, as just noted, is that the 3% was an unusual rate of return that you would not expect to repeat. Note that, instead of using your 5% guess about the future equity premium, you could have used the historical average rate of return on XMKT. From 1927 to 2006, it was 8.9%. In this case, you would have required your project to earn a rate of return of $5\% + 2 \cdot 8.9\% \approx 23\%$.

Now let's choose another project. Let's say you estimate coefficients $a = 3\%$, $b = 0.5$, $c = -1$, $d = 2$, and $e = -2$ for this one. Again, you would need some estimates of the future average rate of return for the four Fama-French factors, just as you needed an estimate for the future average rate of return for the equity premium. For lack of a good source and great intuition, most people just use the historical average rates of return, mentioned above. If you believe the historical averages to be good predictors, you would then estimate your project's appropriate expected rate of return to be

Ok, here comes the real use. The above was just practice.

$$\begin{aligned}\mathcal{E}(\tilde{r}_i) - r_F &= 0.5 \cdot \mathcal{E}(\text{XMKT}) + (-1) \cdot \mathcal{E}(\text{UMD}) + 2 \cdot \mathcal{E}(\text{HML}) + (-2) \cdot \mathcal{E}(\text{SMB}) \\ &= 0.5 \cdot 8.5\% + (-1) \cdot 8.9\% + 2 \cdot 4.6\% + (-2) \cdot 3.8\% \\ &= -3.05\%\end{aligned}$$

With a risk-free rate of return of 5%, you would set a hurdle rate of about $5\% - 3\% = 2\%$.

Some final notes: firm-size and market-beta of firms are sufficiently highly correlated that in most practical capital budgeting applications, you can ignore firm-size and rely on market-beta alone (or the opposite). Moreover, momentum is such a short-term phenomenon that it is irrelevant for long-term capital budgeting purposes. This leaves two factors—the firm's value/growth positioning and its market-beta—as good inputs in a practical capital budgeting model.

Practical notes.

DIG DEEPER

This form of the model does not do justice especially to momentum, which is more of an idiosyncratic effect than a factor exposure to UMD. A better model would work with firms' own characteristics, rather than these factor exposures. (In an APT context, one could then view these characteristics of stocks as picking up firms' betas to some factors. Of course, other researchers believe that these are not really betas, but more a reflection of market inefficiencies, the subject of Chapter 16.) This is all too telegraphic, of course. You should really consult an investments text to learn how to do this better.

In addition, relying on 1-year momentum for a cost of capital estimates for 10-year investments in a corporate context is also not too sensible. This is why UMD is often excluded from this model in a corporate context.

Solve Now!

Q 14.14 Assume that you ran a time-series regression with your project on the Fama-French factors, and found the following:

$$\mathcal{E}(\tilde{r}_i) - r_F = (-2\%) + (1.3) \cdot \text{XMKT} + (0.1) \cdot \text{UMD} + (-1) \cdot \text{HML} + (-0.1) \cdot \text{SMB}$$

What would the Fama-French-Momentum model suggest you use as the hurdle rate for this project?

Deeply Dug Appendix

[No keyterm list for capmtheory-g.]

End of Chapter Problems

Q 14.15 What are the APT factors?

Q 14.16 What are the Fama-French-Momentum factors?

Q 14.17 Assume that you ran a time-series regression with your project on the Fama-French factors, and found the following:

$$\mathcal{E}(\tilde{r}_i) - r_F = (12\%) + (0.3) \cdot \text{XMKT} + (0.3) \cdot \text{UMD} + (-0.5) \cdot \text{HML} + (-0.5) \cdot \text{SMB}$$

What would the Fama-French-Momentum model suggest you use as the hurdle rate for this project?

14 "Solve Now" Answers

1. Because each and every investor purchases an MVE portfolio, the value-weighted market portfolio is MVE.
2. See Formula 14.1
3. It is a measure of risk contribution to an investor's portfolio, if this portfolio is the market portfolio.
4. If the market portfolio lies on the MVE Frontier, then the CAPM holds and all securities must lie on the security markets line.
5. Whether the stock market portfolio is on the MVE Frontier.
6. No.
7. The value-weighted portfolio.
8. Maybe yes, maybe no—but it certainly is *commonly* used as a proxy for the value-weighted domestic stock market portfolio.
9. Because $\mathcal{E}(\tilde{r}_M) > r_F$: the stock market is assumed to have a higher expected rate of return than risk-free Treasury bonds.
10. It has to be the tangency portfolio.
11. Yes, if this stock helps exceptionally well to diversify the market portfolio (beta is negative).
12. See Subsection 14·2.A.
13. The APT is almost like a multi-factor version of the CAPM. Whereas in the CAPM, everything depends on the one factor that is the rate of return on the stock market, in the APT there can be multiple factors—such as the rate of return on the stock market, the rate of return from investing in oil, etc. Unlike the CAPM, the APT does not necessarily assume that the rate of return on the stock market is one factor. It also does not assume that there is an optimal market portfolio, in which all investors should invest.
14. $$\begin{aligned}\mathcal{E}(\tilde{r}_i) - r_F &= 1.3 \cdot \mathcal{E}(\text{XMKT}) + (0.1) \cdot \mathcal{E}(\text{UMD}) + (-1) \cdot \mathcal{E}(\text{HML}) + (-0.1) \cdot \mathcal{E}(\text{SMB}) \\ &= 6.96\% \approx 7\%\end{aligned}$$

    This rate is *above* the risk-free rate, which you would probably want to obtain from the prevailing yield curve.

All answers should be treated as suspect. They have only been sketched and have not been checked.